Imaging of complex media with elastic wave equations

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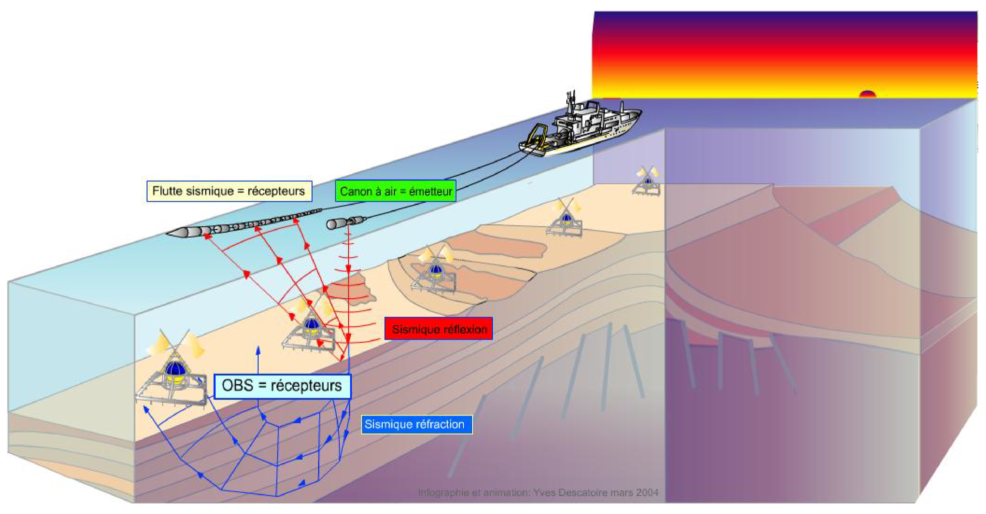
Motivation and context

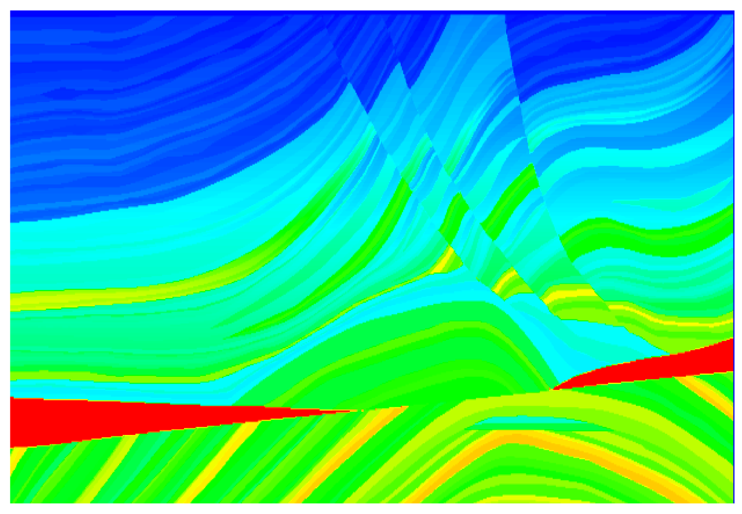
The drilling is expensive, the petroleum industry uses methods able to give images of the area before to do the drilling. From all the famous techniques used by oil companies, the Reverse Time Migration (RTM) is very efficient.

Seismic survey on earth

Seismic survey on sea



Acquisition methods



Speed model

- The seismic acquisition is the artificial generation and the recording of seismic datas.
- The seismic energy is measured by receivers who register seismic signals as waves.
- The RTM works with velocity models of subsurface and allows to do simulations of seismic acquisitions.

Modelling of the problem

Equations

We use acoustic and elastic wave equations to model the problem

$$\begin{cases} \rho(\mathbf{x})\partial_t\mathbf{u}(\mathbf{x},t) + \nabla p(\mathbf{x},t) &= 0, \\ \frac{1}{\mu(\mathbf{x})}\partial_t p(\mathbf{x},t) + \nabla\cdot(\mathbf{u}(\mathbf{x},t)) &= f(\mathbf{x},t) \end{cases} \qquad \begin{cases} \rho(x)\partial_t\mathbf{u}(\mathbf{x},t) = \nabla\cdot\underline{\underline{\sigma}}(\mathbf{x},t), \\ \partial_t\underline{\underline{\sigma}}(\mathbf{x},t) = \underline{\underline{C}}(\mathbf{x})\underline{\underline{\epsilon}}(\overrightarrow{v}(\overrightarrow{x},t)) + f(\mathbf{x},t) \end{cases}$$

where $(\mathbf{x},t)\in\Omega\times[0,T]$

Imaging conditions

We note $\mathbf{u}$ the forward field which is the solution of the previously equations where $f(\mathbf{x},t)$ is the source of the experience, and $\mathbf{d}$ the backward field which is the solution when $f(\mathbf{x},T-t)$ are the data of the receivers. We correlate the forward and backward wavefield to image heterogeneities of the subsurface of the subsurface.

$$I(\mathbf{x}) = \int_0^T \mathbf{u}(\mathbf{x},t)\cdot\mathbf{d}(\mathbf{x},t)dt$$

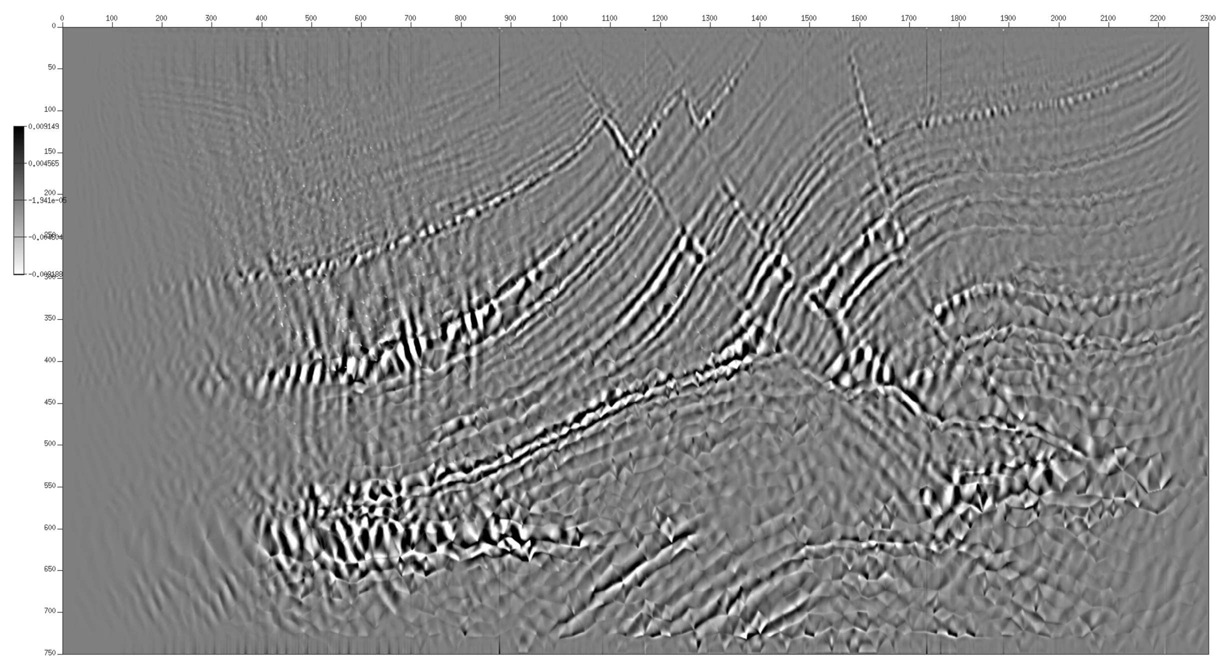
Others formula exist, we can separate the waves P and S :

$$I_{PP}(\mathbf{x}) = \int_0^T [\nabla\cdot\mathbf{u}(\mathbf{x},t)][\nabla\cdot\mathbf{d}(\mathbf{x},t)]dt, \quad I_{SS}(\mathbf{x}) = \int_0^T [\nabla\times\mathbf{u}(\mathbf{x},t)]\cdot[\nabla\times\mathbf{d}(\mathbf{x},t)]dt,$$

The goal is to build new imaging conditions using physical parameters of an elastic media.

Results

This first result is obtained using the first order wave acoustic equation, the velocity model is the marmousi-model described above and we use a discontinuous Galerkin method to solve equations.



Storage strategy

The Griewank algorithm

Several strategies exist to calculate $\mathbf{I}$, the differences are the computation and the storage of $\mathbf{u}^n$.

- Compute $\mathbf{u}^n$ for each $n = N-1,...,0$. This means $O(N^2)$ total steps, which is too expensive for big size problems. In other words, as soon as we need u^n we launch a new simulation, this means too many calculus.
- Record the entire state $\mathbf{u}^n, n = 0,...,N-1$ and access when we need. The cost is only $O(N)$ steps but we need a big memory.
- Record only a given number of propagations, for example all the k iterations and build the field $\mathbf{u}^n$ from 0 to $N-1$ by interpolation. The advantage of this approach is to reduce the implementation cost but it products only an approximate solution.

We have to find a compromise between computation time, precision and storage cost.

- Choose *checkpoints* $\{n_i : i = 0,...,N_C\}$ between 0 and N and give the buffers $\{b_j : j = 1,...,N_B\}$ for the states which must be saved and the states which must be initialized.
- During the **forward** : store u^n at the checkpoint $\{n = n_{i_j} : j = 1,...,N_B\}$ in the buffers.
- During thre **backward** : Calculate $\mathbf{u}^n$ serveral times from the last checkpoint, using the value stored in the buffer $\mathbf{b}_{i_{N_B}}$ to initialize the evolution.
- When the backward step has reached the last checkpoint : $n = n_{i_{N_B}}$, use the buffer $\mathbf{b}_n$ storing the corresponding value.

Example

We consider the following example :

$N = 15, \quad N_B = 3, \quad N_C = 6$. Let I an imaging condition calculing $I(u^n, d^{n+1})$ where u^n is the forward solution and d^n is the backward solution.

We do a first forward et we record 3 checkpoints (0,6 and 11) in buf1, buf2 et buf3 respectively.

We begin the backward at 14. At this step, we need $I(u^{14}, d^{15})$, as in buf3 we have stored u^{11}, all that we need to do is a forward starting from $n = 11$ until 14 (we consider in this case u^{11} as a Cauchy condition)

Now we are at 13. At this step, we need $I(u^{13}, d^{14})$, as in buf3 we have stored u^{11}, all that we need to do is a forward starting from $n = 11$ until 13 (we consider in this case u^{11} as a Cauchy condition)

And so on, the table opposite summarise our example until the time 0.

- The columns bufk are the checkpoints recorded in the buffer k
- The column recomp is the steps of the forward that they will be recomposed
- The numbers in bold are Cauchy conditions for the next forward
- The column n is the u^n used to produce $I(u^n, d^{n+1})$

step	buffer 1	buffer 2	buffer 3	recomposition	n
14	0	6	**11**	12,13,14	14
13	0	6	**11**	12,13	13
12	0	6	**11**	12	12
11	0	**6**	11	7,8	11
10	0	6	**8**	9,10	10
9	0	6	**8**	9	9
8	0	6	8	-	8
7	0	**6**	8	7	7
6	0	6	8	-	6
5	**0**	1	3	1,2,3,4,5	5
4	0	1	**3**	4	4
3	0	1	3	-	3
2	0	**1**	3	2	2
1	0	1	3	-	1
0	0	1	3	-	0

Adjoint state method and RTM

The adjoint state methode is a general method to calculate the gradient of a cost function depending of state variables which are solutions of a direct problem.
J. Tromp (`http://www.princeton.edu/geosciences/tromp/`) and his colleagues define three kernels in this method :

$$K_\rho(\mathbf{x}) = \int_0^T \rho(\mathbf{x})\partial_t\mathbf{s}^\dagger(\mathbf{x},T-t)\cdot\partial_t\mathbf{s}(\mathbf{x},t)dt, K_\mu(\mathbf{x}) = -\int_0^T 2\mu(\mathbf{x})\mathbf{D}^\dagger(\mathbf{x},T-t):\mathbf{D}(\mathbf{x},t)dt, K_\kappa(\mathbf{x}) = -\int_0^T \kappa(\mathbf{x})[\nabla\cdot\mathbf{s}^\dagger(\mathbf{x},T-t)][\nabla\cdot\mathbf{s}(\mathbf{x},t)]dt.$$

$\mathbf{s}^\dagger$ is the adjoint wavefield, $\mathbf{D} = \frac{1}{2}[\nabla\mathbf{s} + (\nabla\mathbf{s})^T] - \frac{1}{3}(\nabla\cdot\mathbf{s})\mathbf{I}$ et $\mathbf{D}^\dagger$ is his adjoint.
We focus on performances of imaging method when we use the kernel :

$$K'_\rho = K_\rho + K_\mu + K_\kappa,$$

This kernel is recognized to reduce numerical artefacts in the case of a global inversion of the earth. But, in seismic migration, the backward wavefield speed d is connected to the displacement of the field s by $d(t) = \partial_t s(t)$ and the forward speed wavefield u is connected to the adjoint wavefield by $u(t) = -\partial_t s^\dagger(T-t)$
In this thesis, we will extend such imaging methods used in seismology to the RTM.